The Schrödinger wave equation

Note Title 1/8/2008

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$\psi(r,t)$ $\quad V(r,t)$

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}$$

Special Case 1 : 1D

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x,t)\psi(x,t) = i\hbar\frac{\partial\psi}{\partial t}$$

Special Case 2: closed system

closed system $\Rightarrow$ energy is conserved

$\Rightarrow V(r,t) = V(r)$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\psi(x,t) = \phi(x)\,T(t)$$

$$-\frac{\hbar^2}{2m}\frac{d^2\phi}{dx^2}T(t)+V(x)\phi(x)T(t)=i\hbar\phi(x)\frac{dT(t)}{dt}$$

$$\underbrace{-\frac{\hbar^2}{2m}\frac{\phi''}{\phi}+V(x)}_{\text{I}}=\underbrace{i\hbar\frac{\dot{T}(t)}{T}=E}_{\text{II}}$$

(I) $$-\frac{\hbar^2}{2m}\frac{\phi''}{\phi}+V=E$$

$$-\frac{\hbar^2}{2m}\frac{d^2\phi}{dx^2}+V\phi=E\phi$$

This is the case in most practical problems.

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\psi(r)+V(r)\psi(r)=E\psi(r)}$$

(II) $$i\hbar\frac{\dot{T}(t)}{T}=E \Rightarrow \dot{T}=-\frac{iE}{\hbar}T$$

$$\Rightarrow T(t)=Ae^{-\frac{iE}{\hbar}t}$$

$$\Rightarrow \psi(x,t)=\phi(x)e^{-\frac{iE}{\hbar}t}$$

$$E=\hbar\omega \Rightarrow \psi(x,t)=\phi(x)e^{-i\omega t}$$

$\Rightarrow$ Wave